Algebra Pre-Assessment (Session 2)

Name SOLUTIONS

Write the letter of the set in the blank next to the corresponding term:

c Integer	a. $\{1, 2, 3, 4, 5, 6, \cdots\}$
a Natural	b. $\{0, 1, 2, 3, 4, 5, 6 \cdots\}$
d Rational	c. $\{\cdots -3, -2, -1, 0, 1, 2, 3, \cdots\}$
e Irrational	d. $\left\{\frac{a}{b}: \textit{for all integers } a \textit{ and } b\ (b \neq 0)\right\}$
b Non-negative	e. $\{\cdots, 1.4142135\ldots, 2.7182818\ldots, 3.1415926\ldots, \cdots\}$

Perform the following operations with integers:

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 213 \\ +188 \\ \hline 401 \end{array}$$

$-8 - (-5) =$

$-8 + 5 = -3$

$14(-12) =$

$10(-12) = -120$

$4(-12) = -48$

-168

$180 \div 5 =$

$5\overline{)180}$ → 36; -15; 30; -30; 0

Perform the following operations with fractions (simplify if possible):

$\frac{2}{5} + \frac{7}{5} = \frac{2+7}{5} = \frac{9}{5}$

$\frac{2}{3} - \frac{1}{2} =$

$4/6 - 3/6 = \frac{4-3}{6} = \frac{1}{6}$

$\frac{4}{3} \cdot \frac{5}{6} = \frac{4 \cdot 5}{3 \cdot 6} = \frac{20}{18}$

$\div 2$

$\frac{10}{9}$

$\frac{6}{5} \div \frac{1}{3} = \frac{6/5}{1/3} \rightarrow \frac{6}{5} \cdot \frac{3}{1} = \frac{18}{5}$

Simplify these expressions as far as possible:

$36 - 4(2^2 + 3)$

$36 - 4(4 + 3)$

$36 - 4(7)$

$36 - 28 = 8$

$4(3 + 2x) - 6(5 - x)$

$4 \cdot 3 + 4 \cdot 2x - 6 \cdot 5 - 6 \cdot (-x)$

$12 + 8x - 30 + 6x$

$14x - 18$ OR $2(7x - 9)$

Circle "T" if the statement is true (valid) or "F" if the statement is false (not valid):

T (F) $3^4 = 64$ $3 \cdot 3 \cdot 3 \cdot 3$; $9 \cdot 9 = 81 \neq 64$

(T) F $|-5| > 0$ $|-5| = 5 > 0$

T (F) 38 is an *odd* number "not divisible by 2" $\frac{38}{2} = 19$

T (F) 51 is a *prime* number "only divisible by itself and 1" $3 \cdot 17 = 51$

(T) F $\frac{0}{k} = 0$, for any $k \neq 0$ zero divided by anything is zero

T (F) $\frac{24}{14} = 1\frac{5}{6}$

$\frac{24}{14} \rightarrow 1\frac{10}{14} = 1\frac{5}{7}$

$1\frac{5}{6} \rightarrow 1\frac{5}{6}$

$1\frac{5}{6} > 1\frac{5}{7}$ not equal

T (F) $\frac{18}{25} = 60\%$ $\frac{18}{25} \cdot 4 = \frac{72}{100} = 72\% \neq 60\%$

T (F) $0.002 = 2 \times 10^3$

$0.002 \rightarrow 2 \times 10^{-3}$

$2 \times 10^3 \rightarrow 2000$

(T) F $\frac{1}{3} = 0.33\overline{3}$

$3\overline{)1.000}$ → $0.333\ldots$; -9; 10; -9; 10; -9; 1

T (F) $(a - b) - c = a - (b - c)$, for any a, b, c

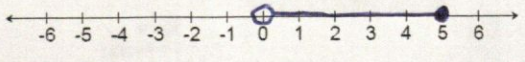
ex. $(10 - 4) - 1 \stackrel{?}{=} 10 - (4 - 1)$

$6 - 1 \stackrel{?}{=} 10 - 3$

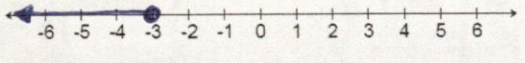
$5 \neq 7$

Draw each interval on the number-line provided:

$(0, 5]$



$-5x - 4 \geq 11$



$-5x - 4 \geq 11$

$+4 \quad +4$

$\frac{-5x}{-5} \geq \frac{15}{-5}$

$x \leq -3$

Algebra Pre-Assessment (page 2)

Simplify the following expressions as far as possible:

$(2^3)^2$ → 2^6 → $(8)^2$ → $8 \cdot 8 = \boxed{64}$

$2 \cdot 2 \cdot 2 = 8$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$(2x^2 + 5x) - (3x + 7)$

$2x^2 + 5x - 3x - 7$

$\boxed{2x^2 + 2x - 7}$

$\frac{ab^{-3}}{a^4b^2}$

$a^{1-4} \cdot b^{-3-2}$

$a^{-3} \cdot b^{-5}$ → $\boxed{\frac{1}{a^3b^5}}$

negative exponents reciprocate

Expand the following polynomials and combine similar terms:

$5(3a - 8)$

$5 \cdot 3a + 5 \cdot (-8)$

$\boxed{15a - 40}$

$(b + 1)^2$

$(b+1)(b+1)$

$b \cdot b + b \cdot 1 + 1 \cdot b + 1 \cdot 1$

$\boxed{b^2 + 2b + 1}$

$(2c - 3)(c^2 + 2c - 5)$

$2c(c^2 + 2c - 5) - 3(c^2 + 2c - 5)$

$2c^3 + 4c^2 - 10c - 3c^2 - 6c + 15$

$\boxed{2c^3 + c^2 - 16c + 15}$

Factor the following polynomials into two binomials. Answers will be of the form (_____)(_____):

$x^2 + 6x + 8$

$\boxed{(x+4)(x+2)}$

factors of 8: 1, 2, 4, 8

sum to 6

$2xy - 8y + 3x - 12$

$2y(x-4) + 3(x-4)$

$\boxed{(2y+3)(x-4)}$

Evaluate this expression using the given values: $2x + y^2$ if $x = 3$ and $y = -4$

$2x + y^2 \Rightarrow 2(3) + (-4)^2 = 6 + (16) = \boxed{22}$

What values of x and y (and z) make the following systems of equations true?

① $2x - y = 6$
② $y + 2 = x$

Substitute ② into ①

$2x - y = 6$ ($x = y+2$)

$2(y+2) - y = 6$

$2y + 4 - y = 6$

$y + 4 = 6$

$-4 \quad -4$

$y = 2$

Substitute $y = 2$ into ②

$y + 2 = x$

$(2) + 2 = x$

$4 = x$

$\boxed{x = 4, y = 2}$

① $2x - 5y + 3z = -1$
② $x + 4y - 2z = 9$
③ $x - 2y - 4z = -5$

Eliminate x with ② and ③

② : $x + 4y - 2z = 9$

$-$③ : $-x + 2y + 4z = +5$

④ $6y + 2z = 14$

Eliminate x with ① and ③

① : $2x - 5y + 3z = -1$

$-2 \cdot$③ : $-2x + 4y + 8z = 10$

⑤ $-y + 11z = 9$

Eliminate y with ④ and ⑤

④ : $6y + 2z = 14$

$6 \cdot$⑤ : $-6y + 66z = 54$

$68z = 68$

$z = 1$

Use $y + z$ in ②:

$x + 4(2) - 2(1) = 9$

$x + 8 - 2 = 9$

$x = 3$

Use z-value to solve for y with ⑤

$-y + 11z = 9$

$-y + 11(1) = 9$

$-y + 11 = 9$

$-11 \quad -11$

$-y = -2$

$y = 2$

$\boxed{x = 3, y = 2, z = 1}$

Algebra Pre-Assessment (page 3)

Find the slope of the following lines.

$4x + y_1 = 0$

$-4x \quad -4x$

$y_1 = -4x$

slope of y_1 is -4

$\frac{-2y_2}{-2} = \frac{8x - 8}{-2}$

$y_2 = -4x + 4$

slope of y_2 is -4

Are the slopes of lines y_1, y_2 (circle your response): *parallel* *perpendicular* *neither*

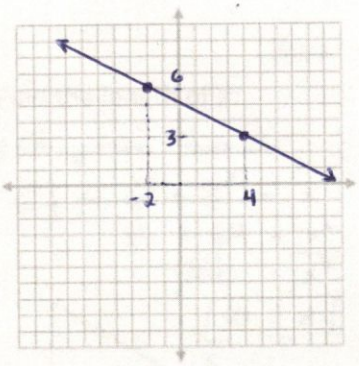
slopes are equal

Find the slope between these two points, and graph the line:

$(-2, 6) \quad (4, 3)$

$x_1, y_1 \quad x_2, y_2$

slope $= \left[\frac{y_1 - y_2}{x_1 - x_2}\right] \quad \frac{6-3}{-2-4} = \frac{3}{-6} = -\frac{1}{2}$



Find the x- and y-intercept(s), then graph the line:

$2x + 3y = -6$

Let $y = 0$: $2x + 3(0) = -6$

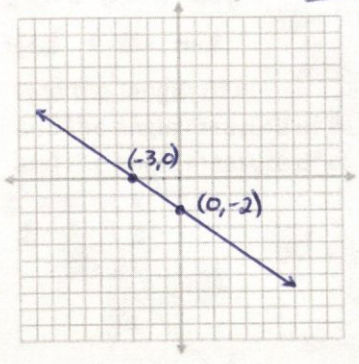
$2x = -6$

$(-3, 0) \leftarrow x = -3$

Let $x = 0$: $2(0) + 3y = -6$

$3y = -6$

$(0, -2) \leftarrow y = -2$



Write the equation of the line with the point $(-2, 4)$ and with a slope of $-\frac{3}{4}$, and graph:

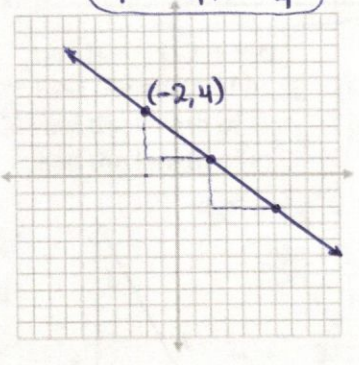
slope-intercept form: $[y - y_1 = m(x - x_1)]$

$y - 4 = -\frac{3}{4}(x - (-2))$

$y - 4 = -\frac{3}{4}x - \frac{6}{4}$

$y = -\frac{3}{4}x - \frac{6}{4} + \frac{16}{4}$

$y = -\frac{3}{4}x + \frac{10}{4}$



If $f(a) = -a^2$ and $g(b) = 3b$, the function $fog = f[g(b)] =$ $-9b^2$ and $fog(2) =$ -36

$-(3b)^2 = -(9b^2)$

$fog(2) = -9b^2$

$= -9(2)^2$

$= -9 \cdot 4 = -36$

Translate the following numerical expression into an English statement:

(Example: $2y \leq 8$ could be written as "Two y is less than or equal to eight")

$6x^2 - 5x - 4 = 0$ "Six x squared minus five x minus four is equal to zero."

(For extra credit, solve the above quadratic equation for x)

$\overset{A}{6}x^2 - \overset{B}{5}x \overset{C}{- 4} = 0$

Quadratic formula: $\left[x = \frac{-B \pm \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A}\right] \rightarrow x = \frac{+5 \pm \sqrt{(-5)^2 - 4 \cdot 6 \cdot (-4)}}{2 \cdot 6} = \frac{5 \pm \sqrt{25 + 96}}{12} = \frac{5 \pm \sqrt{121}}{12}$

$x = \frac{5 \pm 11}{12} \rightarrow \frac{5 - 11}{12} = x = \frac{5 + 11}{12} \rightarrow$ $x = -\frac{1}{2}$, $x = \frac{3}{2}$

Algebra Pre-Assessment (page 4)

Translate each verbal phrase into a numerical expression and solve the problem:

- "To pass algebra, a student must have an exam average of at least 70%. On the first four exams, a student received scores of 82%, 75%, 59%, and 73%. What possible percentages on the final exam would give the student a sufficiently high exam average?

$$\frac{82+75+59+73+X}{5} \geq 70 \longrightarrow \frac{289+X}{5} \geq 70$$

$$289 + X \geq 350 \quad (-289)$$

$$X \geq 61$$

A score of 61% or higher

- "A soccer field has a perimeter of 320 meters. The length between the two goals is 40 meters more than the width between sidelines. What are the dimensions of this soccer field?"

① P (perimeter) = 2 · length + 2 · width = 320

② Length = Width + 40

Substitute ② into ①:

$P = 2 \cdot L + 2 \cdot W$

$= 2(W+40) + 2W$

$= 2W + 80 + 2W$

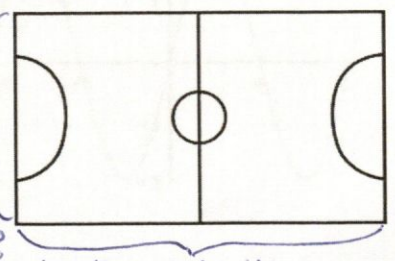
$320 = P = 4W + 80$

$320 = 4W + 80$ $(-80 \quad -80)$

$240 = 4W$

$W = 60$ m

Length = $W + 40$



Find the length of x:

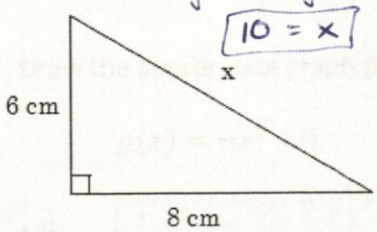
Use pythagorean theorem

$[a^2 + b^2 = c^2] \rightarrow 6^2 + 8^2 = x^2$

$36 + 64 = x^2$

$\sqrt{100} = \sqrt{x^2}$

$10 = x$



Find the length of z:
and the angle measure of b:
and the angle measure of c:

$1^2 + 1^2 = z^2$

$1 + 1 = z^2$

$\sqrt{2} = \sqrt{z^2}$

$z = \sqrt{2}$

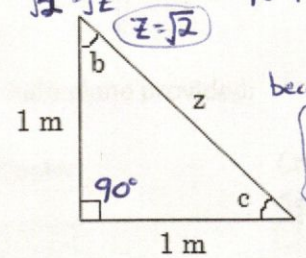
$90° + b + c = 180°$

$b + c = 90°$

because $b = c$

$b = 45°$

$c = 45°$



Find the angle of b: $L = (60) + 40$
and the length of t: $L = 100$
and the length of r:

$90° + 60° + b = 180°$

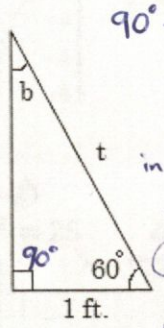
$150 + b = 180$ $(-150 \quad -150)$

$b = 30°$

in a 30-60-90 Δ

$r = 2 \cdot (1 \text{ ft})$

$r = 2 \cdot \text{ft}$



$1^2 + 2^2 = t^2$

$1 + 4 = t^2$

$\sqrt{5} = \sqrt{t^2}$

$t = \sqrt{5}$

Circle the letter (a,b,c,d, e) that corresponds to the one correct response:

$\sqrt{2} \cdot \sqrt{8} =$ _____?

$\sqrt{2} \cdot \sqrt{2 \cdot 2 \cdot 2}$

$= \sqrt{2} \cdot 2\sqrt{2} = 2 \cdot (\sqrt{2} \cdot \sqrt{2})$

$= 2 \cdot (2) = 4$

a. $\sqrt{10}$
b. $2\sqrt{2}$
c. 4
d. $1/4$

$3i(2-5i) =$ _____?
where $i = \sqrt{-1}$

$3i(2-5i)$

$6i - 15i^2$

$*[i^2 = -1]$

$6i - 15(-1)$

$6i + 15$

a. $-9i$
b. $6i - 15$
c. $15 + 6i$
d. -9

$\log_{10}(4) = d$
can also be expressed as _____?

a. $10^d = 4$
b. $4^d = 10$
c. $\ln 4 = d$
d. $e^{10} = 4d$

$[\log_b A = x \rightarrow b^x = A]$

$\log_{10}(4) = d$

$\downarrow$

$10^d = 4$

Algebra Pre-Assessment (page 5)

Write the letter of each graph in the blank next to the corresponding function:

d $f(x) = 2\sin(x)$

b $f(x) = 2\cos(x)$

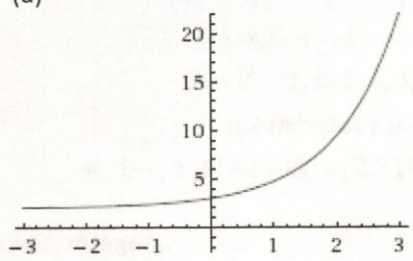
f $f(x) = \tan(x)$

c $f(x) = x^3 + 2$

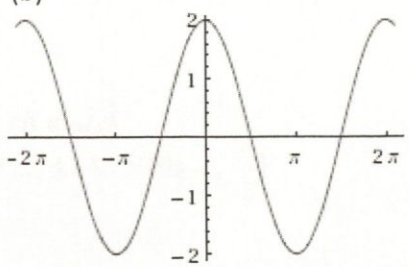
a $f(x) = e^x + 2$

e $f(x) = \ln(x) + 2$

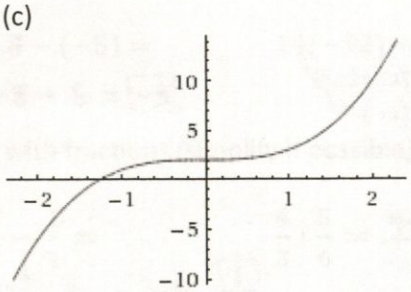
(a)



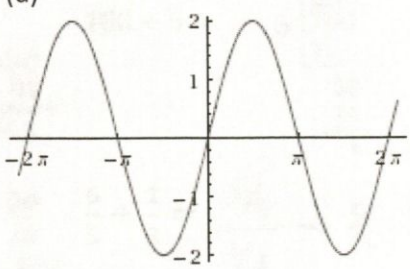
(b)



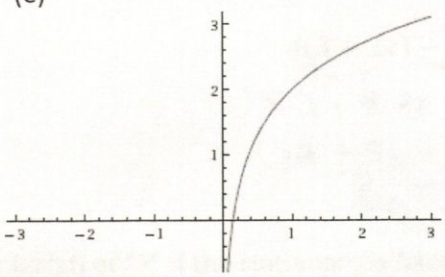
(c)



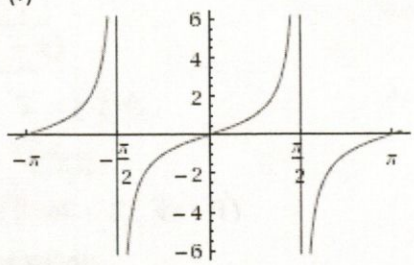
(d)



(e)



(f)



Draw the appropriate graph on the coordinate plane provided:

$g(x) = -x^2 + 4$ a parabola

Let $x=0$

$g(0) = -(0)^2 + 4$

$g(0) = 4$

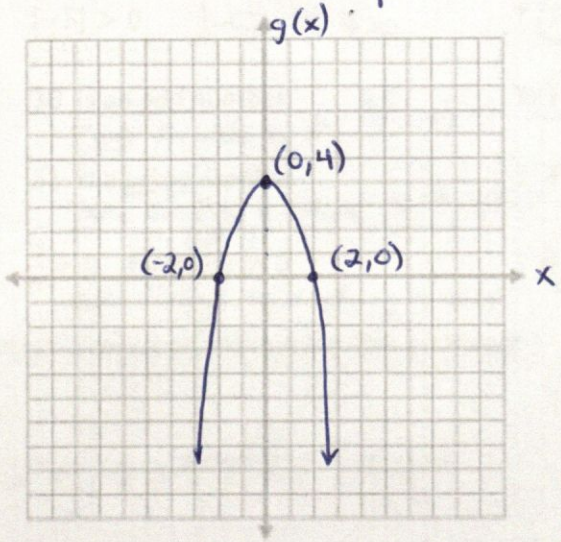
Let $g(x)=0$

$0 = -x^2 + 4$

$-4 = -x^2$

$\sqrt{4} = \sqrt{x^2}$

$\pm 2 = x$



center at (3, -2)

$(x-3)^2 + (y+2)^2 = 25$ a circle

right 3 left 2 $\sqrt{25} = 5$ radius